Journal of Engineering Mathematics, Vol. 15, No. 1, January 1981

Effect of stratification on hydrodynamic pressures on dams

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(Received March 10, 1980)

SUMMARY

The effect of stratification of the fluid in the reservoir on hydrodynamic pressures on dams due to horizontal, harmonic ground accelerations has been analyzed. It has been found that both the zeroth-order solution, which corresponds to the constant-density solution, and the first-order solution have two components in the hydrodynamic pressure distribution, an in-phase component and an out-of-phase component which is 90° lagging. The out-of-phase components vanish in the absence of surface waves, and they become dominant when the wave-effect parameter C becomes large. The wave-effect parameter C is defined as $g/(\omega^2 h)$, where g is the gravitational constant, ω the oscillation frequency and h the height of the fluid in the reservoir. The total horizontal force on a dam due to harmonic ground excitations has also been presented.

1. Introduction

An important factor in the design of dams in seismic regions is the accurate determination of the hydrodynamic pressure exerted on the upstream face of a dam during earthquakes. This factor becomes increasingly critical if the dam under consideration is located on the upstream side of a densely populated community.

During an earthquake a dam, through its interaction with the foundation and the abutments, accelerates into and away from the water in the reservoir, and as a result, the water exerts a hydrodynamic pressure, in excess of the hydrostatic pressure, on the dam surface. The period of the ground excitation, T, during a typical earthquake may range from 0.1 sec to 10 seconds, and the amplitude of vibration is of the order of 1 ft. Therefore the velocity of a fluid particle in the reservoir, which is of the order of ωa with ω $(=2\pi/T)$ being the vibration frequency and a the maximum amplitude or displacement of vibration of a dam, is very small in comparison with the speed of sound V_s in water (about 4720 ft/sec). Realistic values of the ratio $\omega a/V_s$ range from 10^{-2} to 10^{-4}. Hence we may regard the water in the reservoir as an incompressible fluid.

For an infinitely long reservoir, Westergaard [10] first derived an expression for the hydrodynamic pressure exerted on a two-dimensional dam with vertical upstream face by an incompressible, inviscid fluid in the reservoir as a result of horizontal, harmonic ground motion in a direction perpendicular to the dam. He found that this hydrodynamic pressure is the same as if a certain body of fluid, often called the 'added mass', was forced to move back and forth with the dam. In a discussion to Westergaard's [10] paper, Von Kármán [5] presented a simple momentum-balance method and obtained a distribution of the added mass, consequently the hydrodynamic pressure, along the vertical upstream face of a rigid dam, very close to Westergaard's results.

Since the pioneering work of Westergaard, a series of investigations has been conducted to study the hydrodynamic effect on concrete dams for incompressible and compressible water. Kotsubo [6, 7] obtained a general solution for both transient and steady-state hydrodynamic pressure acting on a rigid concrete dam. Chopra [1] demonstrated that the hydrodynamic response of compressible water during an earthquake is somewhat different from that due to incompressible water.

For a dam whose upstream face is not vertical such as an earth dam, Zanger [12] and Zanger & Haefeli [13] determined the hydrodynamic pressures experimentally using an electrical analogue. Recently, Chwang & Housner [4] solved analytically the two-dimensional problem of the added-mass effect due to a horizontal acceleration of a rigid dam with an inclined upstream face of constant slope by adopting the generalized Von Kármán [5] momentum-balance approach. They discovered that the normal force coefficient remains practically constant at around 0.5 for all slopes. In a subsequent paper, Chwang [2] presented an integral solution for the earthquake force on a rigid, sloping dam based on the exact, two-dimensional potential-flow theory. His results were compared with those derived from the momentum-balance method, and the two methods were found to be in reasonable agreement, especially for the total force exerted on the face of the dam.

The effect of finite reservoir on the hydrodynamic pressure was investigated by Werner & Sundquist [9] and by Chwang [3]. Chwang [3] found that for horizontal accelerations the hydrodynamic pressure force decreases as the size of the reservoir decreases. He also found that the effect of vertical acceleration on the pressure force on a dam is simply to adjust the hydrostatic pressure by replacing the gravitational constant by an effective gravitational acceleration and this is true for any arbitrary shapes of the reservoir.

When a rigid dam accelerates away from the water, the resulting hydrodynamic pressure on its upstream face would become negative. Should this pressure become negatively so large that the total absolute pressure (hydrodynamic plus hydrostatic and atmospheric pressure) is less than the critical cavitation pressure, cavitation could take place on the dam surface. The possibility of cavitation at some point on the upstream face of a dam has been discussed by Chwang [3] and by Mei et. al. [8].

Every stream flowing into the reservoir carries some suspended sediment. Due to gravitational settling or due to the temperature variation, the density of the fluid in the reservoir is often stratified. The objective of this paper is to analyze the effect of stratification on hydrodynamic pressures on dams due to horizontal, harmonic ground accelerations. The dams are assumed to be rigid and to have vertical upstream faces. Depending on the period of the ground excitation T and the depth of fluid in the reservoir h, surface waves may also play an important role in the determination of hydrodynamic pressures. If $T = 10$ sec and $h = 300$ ft, then the wave-effect parameter C, which is a measure of relative importance of gravitational surface wave effect to the inertial effect due to vibration and which is defined by $C = g/(\omega^2 h)$, is about 0.27 where $\omega = 2\pi/T$ and $g = 32.2$ ft/sec^2. Hence the presence of surface waves is quite important in determining the resulting hydrodynamic pressures on dams. The surface-wave effect has also been analyzed in this paper.

2. Governing equations

Let us consider a dam with a vertical upstream face. The y axis points vertically upwards and the x axis is perpendicular to the y axis in the horizontal plane and measured from the upstream face of the dam (see Figure 1). The reservoir bottom is the plane $y = 0$ and the undisturbed water surface is at $y = h$. The dam is assumed to be rigid and subject to a harmonic ground acceleration of $-a\omega^2 \sin \omega t$ in the x direction. The displacement and velocity of the dam in the x direction corresponding to this ground acceleration are $a \sin \omega t$ and $a\omega \cos \omega t$ respectively. The maximum displacement of the dam, a, is assumed to be small, so is the deviation of the free surface from its undisturbed level, $\eta(x,t)$.

Since the fluid in the reservoir is assumed to be incompressible and inviscid, the continuity equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{1}$$

and the linearized incompressibility condition is

$$\frac{\partial \rho'}{\partial t} + v\,\frac{d\bar{\rho}}{dy} = 0\,, \tag{2}$$

where u and v are the velocity components in the x and y directions respectively, $\bar{\rho}$ is the mean density which is a function of y only, and ρ' is the density perturbation. The linearized equations of motion are (see Yih, [11])

$$\bar{\rho}\,\frac{\partial u}{\partial t} = -\frac{\partial p'}{\partial x}\,, \tag{3}$$

$$\bar{\rho}\,\frac{\partial v}{\partial t} = -\frac{\partial p'}{\partial y} - \rho' g\,, \tag{4}$$

where p' is the pressure perturbation or the hydrodynamic pressure due to ground acceleration and g is the gravitational constant. The mean pressure or the hydrostatic pressure is given by

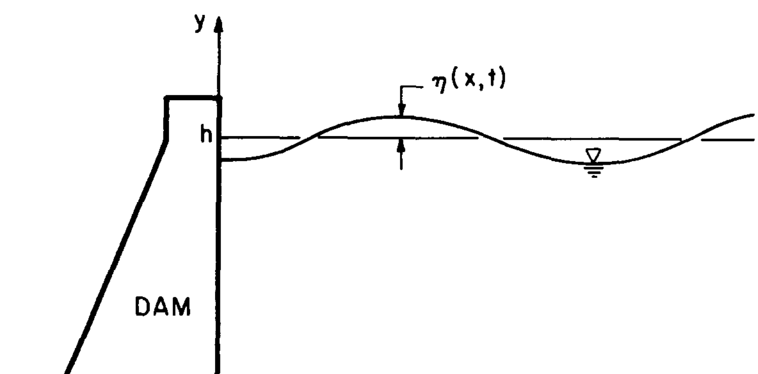

Figure 1. Schematic diagram of a dam-reservoir system.

$$\bar{p} = -\int_h^y \bar{\rho} g dy \tag{5}$$

such that the mean pressure vanishes at the undisturbed water surface.

By introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \tag{6}$$

equation (1) is satisfied automatically. Since there are surface waves propagating away from the dam due to its oscillation, we assume the stream function to be of the form

$$\psi = f(y) e^{i(kx - \omega t)}, \tag{7}$$

where $k (= 2\pi/\lambda$, λ being the wavelength) is the wave number. If ρ' has the same time dependence as the stream function ψ, equation (2) gives

$$i\omega\rho' = v \frac{d\bar{\rho}}{dy}. \tag{8}$$

Eliminating p' from equations (3) and (4), and substituting (6), (7), and (8) into the resulting equation, we obtain

$$\frac{d}{dy}\left(\bar{\rho} \frac{df}{dy}\right) - k^2 \left(\bar{\rho} + \frac{g}{\omega^2} \frac{d\bar{\rho}}{dy}\right) f = 0. \tag{9}$$

Equation (9) is the governing equation for $f(y)$.

At the reservoir bottom the vertical velocity v vanishes. Hence, by (6) and (7), the boundary condition at $y = 0$ is

$$f(0) = 0. \tag{10}$$

On the free surface we require the pressure to vanish. Thus

$$\bar{p} + p' = 0 \quad \text{at} \quad y = h + \eta(x, t). \tag{11}$$

By (3), (6) and (7), we have

$$p' = (\omega/k) \bar{\rho} f'(y) e^{i(kx - \omega t)}, \tag{12}$$

where $f'(y)$ denotes df/dy. Since there is no mean velocity in the x direction, the kinematic boundary condition on the free surface is

$$v = \frac{\partial \eta}{\partial t} \quad \text{at} \quad y = h + \eta(x, t). \tag{13}$$

By (5), (11), (12), and (13), the linearized boundary condition on the free surface becomes

$$f'(h) - g(k/\omega)^2 f(h) = 0. \tag{14}$$

On the upstream face of the dam, $x = 0$, we require

$$f'(y) = a\omega \quad \text{at} \quad x = 0 \,. \tag{15}$$

We shall now seek solutions of equation (9) satisfying boundary conditions (10), (14) and (15) for a given mean density of the fluid in the reservoir, $\bar{\rho}(y)$.

3. Constant density solution

Before we discuss the solution for a stratified fluid, we first study a simple case where the density of the fluid is a constant,

$$\bar{\rho}(y) = \rho_0, \text{ a constant.} \tag{16}$$

For constant density ρ_0, equation (9) reduces to

$$f'' - k^2 f = 0 \,, \tag{17}$$

where f'' denotes d^2f/dy^2. The complete solution of (17), which includes both positive and negative values of k^2 and which satisfies the boundary conditions (10) and (14), yields a stream function ψ in the form of

$$\psi = A_0 \sinh k_0 y e^{i(k_0 x - \omega t)} + \sum_{n=1}^{\infty} A_n \sin k_n y \, e^{-k_n x} e^{-i\omega t}, \tag{18}$$

where k_0 satisfies

$$\cosh k_0 h - C k_0 h \sinh k_0 h = 0 \,, \tag{19}$$

k_n satisfies

$$\cos k_n h + C k_n h \sin k_n h = 0 \qquad (n = 1, 2, 3, \ldots), \tag{20}$$

$$C = g/(\omega^2 h) \,, \tag{21}$$

and A_0 and A_n $(n = 1, 2, 3, \ldots)$ are arbitrary constants. We note that (19) and (21) give the usual dispersion relation for surface waves,

$$\omega^2 = g k_0 \tanh k_0 h \,. \tag{22}$$

Since the frequency of the ground excitation, ω, is given, equation (22) determines uniquely the wave number k_0 and consequently the wavelength $\lambda_0 (= 2\pi/k_0)$ of the surface wave produced by the vibration of the dam.

By equations (7), (15) and (18), we obtain the constants A_0 and A_n as

$$A_0 = \frac{2a\omega P_0}{k_0{}^2 h(1 + CP_0{}^2)}, \tag{23a}$$

$$A_n = \frac{2a\omega P_n}{k_n{}^2 h(1 - CP_n{}^2)} \quad (n = 1,2,3, ...), \tag{23b}$$

where

$$P_0 = \sinh k_0 h \text{ and } P_n = \sin k_n h \quad (n = 1,2,3,...). \tag{23c}$$

The pressure p' may be obtained from equations (3), (6) and (18) as

$$p' = \rho_0 \omega\, [A_0 \cosh k_0 y e^{i(k_0 x - \omega t)} - i \sum_{n=1}^{\infty} A_n \cos k_n y e^{-k_n x} e^{-i\omega t}]. \tag{24}$$

Taking the real part of (24), we obtain the hydrodynamic pressure distribution on the upstream face of the dam at $x = 0$ as

$$\frac{p'(y)}{\rho_0 h(-a\omega^2)} = C_{p0} \sin \omega t + C_{q0} \cos \omega t, \tag{25}$$

where the in-phase (with respect to the given ground harmonic acceleration $-a\omega^2 \sin \omega t$) pressure coefficient C_{p0} is given by

$$C_{p0} = 2 \sum_{n=1}^{\infty} \frac{P_n \cos k_n y}{k_n{}^2 h^2 (1 - CP_n{}^2)} \quad (0 \leqslant y \leqslant h), \tag{26}$$

and the out-of-phase (90° lagging) pressure coefficient C_{q0} is given by

$$C_{q0} = -\frac{2P_0 \cosh k_0 y}{k_0{}^2 h^2 (1 + CP_0{}^2)} \quad (0 \leqslant y \leqslant h). \tag{27}$$

In Figure 2, the pressure coefficient C_{p0} is plotted versus the vertical distance y/h for several values of the wave-effect parameter C. The parameter C obtained by (21) is a direct measure of the gravity effect to the inertial effect due to oscillation. A small value of C means that the gravity effect is negligible in calculating the hydrodynamic pressure. On the other hand, for large values of C, the gravity effect becomes important. Thus the surface waves caused by the oscillation of the dam must be taken into account. We note from Figure 2 that at a fixed depth of y/h, the hydrodynamic pressure decreases as C increases. Physically it means that surface waves become important as C increases and energy is radiated by waves propagating away from the dam. When C vanishes, there is no gravity effect on the hydrodynamic pressure, and surface waves cease to exist. Thus the pressure distribution becomes the same as that given by Westergaard [10] and Chwang [2] at $C = 0$. For fixed values of C, the hydrodynamic pressure, as seen in Figure 2, increases as the height y/h decreases and attaines a maximum value of C_{p0} at the bottom of the reservoir $y = 0$.

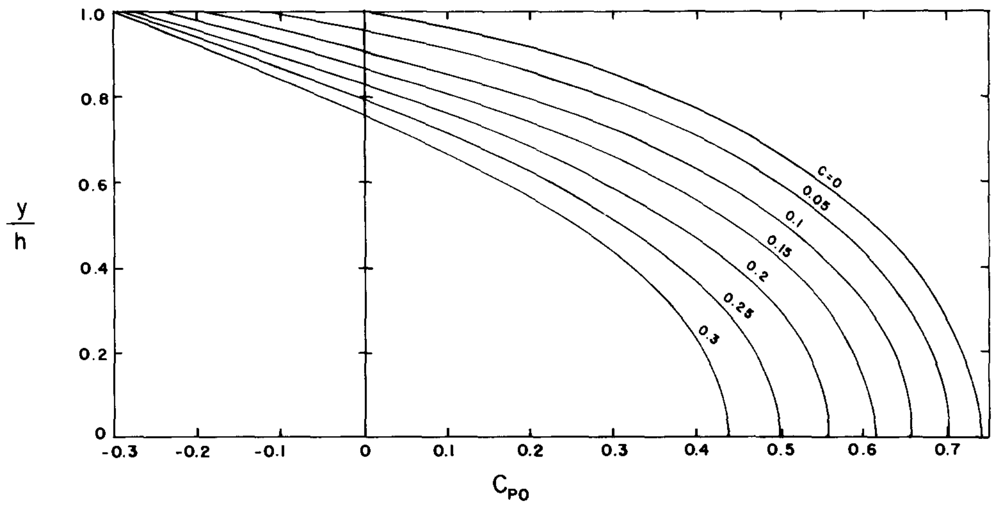

Figure 2. The in-phase pressure distribution on the upstream face of a dam for various values of $C = g/(\omega^2 h)$.

The out-of-phase pressure coefficient C_{q0} is plotted in Figure 3 versus the vertical height y/h for several fixed values of C from 0.05 to 0.4. For fixed values of y/h, the magnitude of C_{q0} increases as C increases; and for fixed values of C, it increases as y/h increases (approaches to the undisturbed free surface $y/h = 1$) as it should be since the out-of-phase component of the hydrodynamic pressure is entirely due to the presence of surface waves. When $C = 0, C_{q0}$ vanishes since there are no surface waves then. Comparing Figure 2 with Figure 3, we see that the magnitude of C_{q0} is comparable to that of C_{p0}. Therefore we cannot neglect the surface-wave effect on the hydrodynamic pressure due to earthquakes unless C is very small.

The total hydrodynamic pressure force on the dam can be found by integrating equation (25) as

$$F = \int_0^h p'(y)dy = -\rho_0 h^2 a\omega^2 (C_{F0} \sin \omega t + C_{L0} \cos \omega t), \tag{28}$$

where

$$C_{F0} = 2 \sum_{n=1}^{\infty} \frac{P_n^{\,2}}{k_n^{\,3} h^3 (1 - CP_n^{\,2})}, \tag{29}$$

and

$$C_{L0} = -\frac{2P_0^{\,2}}{k_0^{\,3} h^3 (1 + CP_0^{\,2})}. \tag{30}$$

In Figure 4 both the in-phase force coefficient C_{F0} and the out-of-phase force coefficient C_{L0} are plotted versus the parameter C ranging from 0 to 0.5. At $C = 0$, the out-of-phase component vanishes and the in-phase force coefficient C_{F0} equals to 0.543 which is precisely the value given by Westergaard [10] neglecting the surface-wave effect. As the value of C increases, C_{F0} decreases monotonically while the magnitude of C_{L0} increases. At $C = 0.5$, C_{F0} has a value of 0.068 and C_{L0} becomes -0.4.

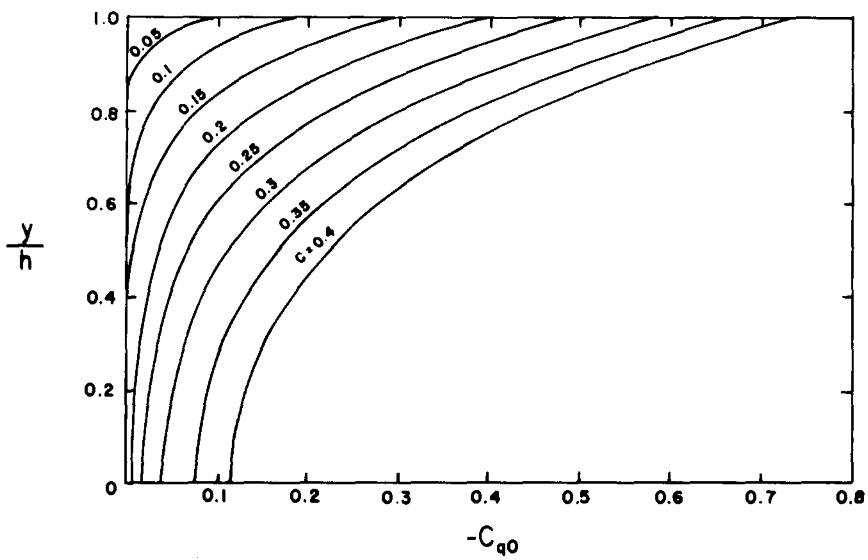

Figure 3. The out-of-phase pressure distribution on the upstream face of a dam for various values of $C = g/(\omega^2 h)$.

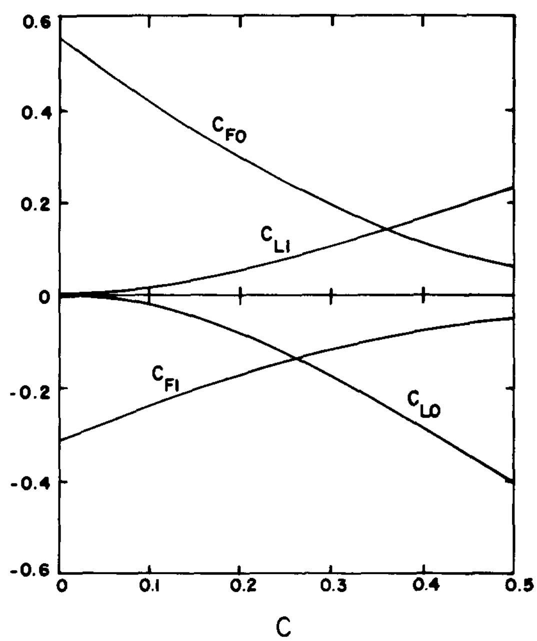

Figure 4. The zeroth-order in-phase and out-of-phase force coefficients C_{F0} and C_{L0}, and the first-order in-phase and out-of-phase force coefficients C_{F1} and C_{L1} for a slightly stratified fluid versus the wave-effect parameter $C = g/(\omega^2 h)$.

4. Solution for a slightly stratified fluid

If the mean density of fluid in the reservoir is not constant but a linear function of y given by

$$\bar{\rho}(y) = \rho_0(1 - \epsilon y/h) \quad (0 < \epsilon \ll 1), \tag{31}$$

where ρ_0 is the density of fluid at the reservoir bottom, we assume that the function $f(y)$ and the wave number k defined in (7) can be expanded in terms of ϵ as

$$k = k_0 + \epsilon \ell_0 + \ldots\ldots , \tag{32a}$$

$$f(y) = f_0 + \epsilon f_1 + \ldots\ldots . \tag{32b}$$

Substituting (31) and (32) into (9) and collecting all ϵ° terms, we have

$$f_0'' - k_0^2 f_0 = 0 . \tag{33}$$

Collecting all terms of the order of $O(\epsilon)$, we obtain a differential equation for f_1,

$$f_1'' - k_0^2 f_1 = f_0'/h + k_0 (2\ell_0 - k_0 C) f_0 , \tag{34}$$

where C is again defined by equation (21). The boundary condition requires that

$$f_0(0) = 0 \quad \text{and} \quad f_1(0) = 0. \tag{35}$$

Substituting (32) into (14), we have

$$f_0' - Ck_0^2 h f_0 = 0 \quad \text{at} \quad y = h, \tag{36a}$$

and

$$f_1' - Ck_0^2 h f_1 = 2Ck_0 \ell_0 h f_0 \quad \text{at} \quad y = h. \tag{36b}$$

On the upstream face of the dam, (15) reduces to

$$f_0'(y) = a\omega \quad \text{and} \quad f_1'(y) = 0 \quad \text{at} \quad x = 0. \tag{37}$$

The complete solution of (33), including both positive and negative eigenvalues of k_0^2, satisfying boundary conditions (35) and (36a) gives a stream function ψ which is exactly the same as that given by equation (18). We note that the positive eigenvalue of k_0^2 gives a real eigenvalue of k_0 which satisfies condition (19). However, the negative eigenvalues of k_0^2 produce a set of infinitely many imaginary eigenvalues ik_n ($n = 1,2,3 \ldots\ldots$) which satisfy equation (20). The coefficients A_0 and A_n ($n = 1,2,3 \ldots\ldots$) in (18) are given by equations (23a) to (23c) after applying the boundary condition (37). Therefore the zeroth-order solution is exactly the same as that given in the previous section. The in-phase and out-of-phase pressure coefficients C_{p0}

and C_{q0} given by equations (26) and (27) are shown graphically in Figures 2 and 3 respectively. The corresponding force coefficients C_{F0} and C_{L0} are shown in Figure 4.

To find the first-order solution for $f_1(y)$, we note that for positive values of $k_0{}^2$ the solution of equation (34) satisfying the boundary condition (35) is

$$f_1{}^+ = B_0 \sinh k_0 y + \tfrac{1}{2} A_0 \, [(y/h) \sinh k_0 y + (2\ell_0 - k_0 C) y \cosh k_0 y], \tag{38}$$

where k_0 is given by (19) and A_0 given by (23a). Applying equation (36b), we can obtain the value of ℓ_0 as

$$\ell_0 = \frac{1 - Ck_0{}^2h^2(1 + C - C^2k_0{}^2h^2)}{2k_0h^2(C - 1 + C^2k_0{}^2h^2)} . \tag{39}$$

For negative values of $k_0{}^2$ which yield a set of infinitely many imaginary eigenvalues ik_n ($n = 1,2,3 \ldots\ldots$), the solution of equation (34) satisfying the boundary condition (35) is

$$f_1^- = \sum_{n=1}^{\infty} \{B_n \sin k_n y + \tfrac{1}{2} A_n \, [(y/h) \sin k_n y + (2\ell_n - k_n C) y \cos k_n y]\}, \tag{40}$$

where the k_n's are given by (20) and the A_n's by (23b). The $i\ell_n$'s are the imaginary counterparts of ℓ_0 and they are determined by means of condition (36b) as

$$\ell_n = \frac{1 + Ck_n{}^2h^2(1 + C + C^2k_n{}^2h^2)}{2k_nh^2(1 - C + C^2k_n{}^2h^2)} \quad (n = 1,2,3 \ldots\ldots). \tag{41}$$

Therefore the stream function ψ now becomes

$$\psi = [A_0 \sinh k_0 y + \epsilon f_1{}^+ + \ldots] \, e^{i(k_0 + \epsilon\ell_0 + \ldots)x - i\omega t} + [\sum_{n=1}^{\infty} A_n \sin k_n y + \epsilon f_1^- + \ldots] \, e^{-(k_n + \epsilon\ell_n + \ldots)x - i\omega t} . \tag{42}$$

Applying the boundary condition at the upstream face of the dam, (37), and noting that $\cosh k_0 y$ and $\cos k_n y$ ($n = 1,2,3 \ldots\ldots$) form a set of orthogonal functions over the interval from $y = 0$ to $y = h$, we obtain the coefficients B_0 and B_n in (42) as

$$B_0 = \tfrac{1}{2} A_0 (C - 2\ell_0/k_0) - [k_0 h (1 + CP_0{}^2)]^{-1} \{\tfrac{1}{2} A_0 \, [P_0{}^2/(k_0 h) + 2k_0 h F_{00} + 2k_0 h^2 (2\ell_0 - k_0 C) E_{00}] + \sum_{m=1}^{\infty} A_m \, [D_{m0} + k_m h F_{0m} + k_m h^2 (k_m C - 2\ell_m) E_{m0}]\} , \tag{43a}$$

$$B_n = \tfrac{1}{2} A_n \,(C - 2\ell_n/k_n) - [k_n h(1 - CP_n{}^2)]^{-1} \,\{A_0\,[D_{0n} + k_0 hF_{0n} + k_0 h^2(2\ell_0 - k_0 C)E_{0n}] + \sum_{m=1}^{\infty} A_m\,[D_{mn} + k_m hF_{mn} + k_m h^2(k_m C - 2\ell_m)E_{mn}]\} \qquad (n = 1,2,3\,.....)\,, \tag{43b}$$

where for $m = 1,2,3\,.....$ and $n = 1,2,3\,.....$

$$D_{0n} = \frac{k_0(Q_0Q_n - 1) + k_nP_0P_n}{(k_0{}^2 + k_n{}^2)h}\,, \tag{44a}$$

$$D_{n0} = \frac{k_0P_0P_n - k_n(Q_0Q_n - 1)}{(k_0{}^2 + k_n{}^2)h}\,, \tag{44b}$$

$$D_{mn} = \frac{k_m(1 - Q_mQ_n) - k_nP_mP_n}{(k_m{}^2 - k_n{}^2)h} \qquad (m \neq n), \tag{44c}$$

$$= \frac{P_n{}^2}{2k_nh} \qquad (m = n), \tag{44d}$$

$$E_{00} = \frac{k_0h(P_0{}^2 + Q_0{}^2) - P_0Q_0}{4k_0{}^2h^2}\,, \tag{45a}$$

$$E_{0n} = \frac{k_0Q_0Q_n + (1 - C)\,k_nP_0P_n}{(k_0{}^2 + k_n{}^2)\,h}\,, \tag{45b}$$

$$E_{n0} = \frac{(1 - C)\,k_0P_0P_n - k_nQ_0Q_n}{(k_0{}^2 + k_n{}^2)\,h}\,, \tag{45c}$$

$$E_{mn} = \frac{(C - 1)\,k_nP_mP_n - k_mQ_mQ_n}{(k_m{}^2 - k_n{}^2)\,h} \qquad (m \neq n), \tag{45d}$$

$$= \frac{k_nh\,(P_n{}^2 - Q_n{}^2) + P_nQ_n}{4k_n{}^2h^2} \qquad (m = n), \tag{45e}$$

$$F_{00} = \frac{k_0{}^2h^2 + 2k_0hP_0Q_0 - P_0{}^2}{4k_0{}^2h^2}\,, \tag{46a}$$

$$F_{0n} = \frac{(k_0{}^2 - k_n{}^2)\,(1 - Q_0Q_n) - 2k_0k_nP_0P_n}{(k_0{}^2 + k_n{}^2)^2h^2}\,, \tag{46b}$$

$$F_{mn} = \frac{(k_m{}^2 + k_n{}^2)\,(Q_mQ_n - 1) + 2k_mk_nP_mP_n}{(k_m{}^2 - k_n{}^2)^2h^2} \qquad (m \neq n), \tag{46c}$$

$$= \frac{k_n{}^2h^2 + 2k_nhP_nQ_n - P_n{}^2}{4k_n{}^2h^2} \qquad (m = n), \tag{46d}$$

and

$$P_0 = \sinh k_0h, \qquad P_n = \sin k_nh, \tag{23c}$$

$$Q_0 = \cosh k_0h, \qquad Q_n = \cos k_nh. \tag{23d}$$

The hydrodynamic pressure p' on the upstream face of the dam at $x = 0$ may be obtained from equations (3), (6) and (42), thus

$$p' = \omega\rho_0(1 - \epsilon y/h)\{(k_0 + \epsilon\ell_0 + ...)^{-1}(k_0A_0\cosh k_0y + \epsilon df_1^+/dy + ...)\cos\omega t - \sum_{n=1}^{\infty}[(k_n + \epsilon\ell_n + ...)^{-1}(k_nA_n\cos k_ny + \epsilon df_1^-/dy + ...)\sin\omega t]\}, \tag{47a}$$

or in dimensionless form

$$\frac{p'}{\rho_0h(-a\omega^2)} = (C_{p0} + \epsilon C_{p1} + ...)\sin\omega t + (C_{q0} + \epsilon C_{q1} + ...)\cos\omega t. \tag{47b}$$

The zeroth-order in-phase and out-of-phase pressure coefficients C_{p0} and C_{q0} are given by

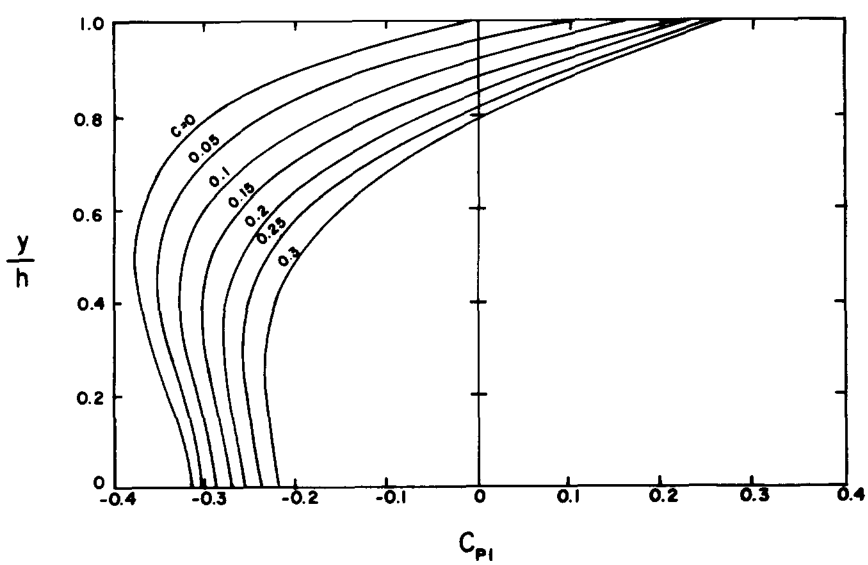

Figure 5. The first-order in-phase pressure distribution on the upstream face of a dam for various values of $C = g/(\omega^2h)$.

equations (26) and (27) respectively. The first-order in-phase pressure coefficient C_{p1} is given by

$$2a\omega h C_{p1} = \sum_{n=1}^{\infty} \{(2B_n - CA_n)\cos k_n y + A_n \left[\frac{\sin k_n y}{k_n h} - \frac{y \cos k_n y}{h} + (k_n C - 2\ell_n) y \sin k_n y\right]\}, \tag{48}$$

where A_n is determined by (23b), B_n by (43b) and ℓ_n by (41). Figure 5 shows the numerical values of C_{p1} as determined by (48) versus the height y/h for fixed values of C from $C = 0$ to $C = 0.3$. We note from Figure 5 that for fixed values of C, C_{p1} is negative at the reservoir bottom ($y = 0$). It decreases further as y/h increases until it reaches a minimum value, then it increases as y/h increases. At the undisturbed free surface $y = h$, the pressure coefficient C_{p1} becomes positive except the case of $C = 0$ in which C_{p1} vanishes. For fixed values of y/h, C_{p1} increases as C increases as a result of the surface-wave effect. However, due to stratification of the fluid in the reservoir, C_{p1} remains negative over a large range of height y/h except the region close to the free surface.

The first-order out-of-phase pressure coefficient C_{q1} in (47b) is given by

$$2a\omega h C_{q1} = (CA_0 - 2B_0)\cosh k_0 y - A_0 \left[\frac{\sinh k_0 y}{k_0 h} - \frac{y \cosh k_0 y}{h} + (2\ell_0 - k_0 C) y \sinh k_0 y\right], \tag{49}$$

where A_0 is given by (23a), B_0 by (43a) and ℓ_0 by (39). The numerical values of C_{q1} are plotted in Figure 6 versus the vertical height y/h for several fixed values of C from $C = 0.05$ to $C = 0.4$.

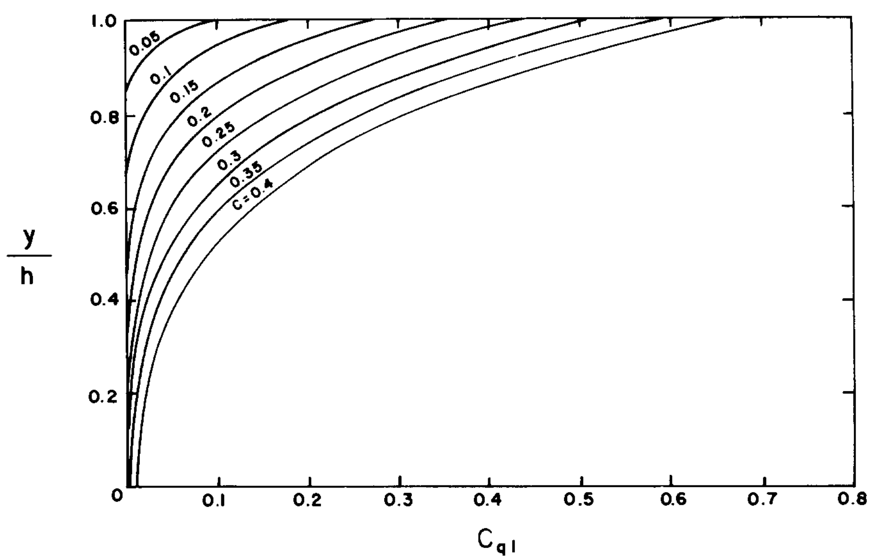

Figure 6. The first-order out-of-phase pressure distribution on the upstream face of a dam for various values of $C = g/(\omega^2 h)$.

For $C = 0$, C_{q1} is identically equal to zero. As C increases, the wave effect becomes important, hence C_{q1} increases for fixed height of y/h. For any given values of C, C_{q1} increases with increasing height y/h, it reaches a maximum value at the undisturbed free surface $y = h$.

The total hydrodynamic pressure force on the dam may be obtained by integrating equation (47b) to give

$$F = \int_0^h p'(y)dy = -\rho_0 h^2 a\omega^2 \, [(C_{F0} + \epsilon C_{F1} + \ldots) \sin \omega t$$
$$+ (C_{L0} + \epsilon C_{L1} + \ldots) \cos \omega t], \tag{50}$$

where the zeroth-order in-phase and out-of-phase force coefficients C_{F0} and C_{L0} are given by equations (29) and (30) respectively. The first-order in-phase force coefficient C_{F1} may be obtained from (48) as

$$2a\omega h C_{F1} = \sum_{n=1}^{\infty} \{[2B_n - (1 - 2C - C^2 k_n{}^2 h^2) A_n] \frac{P_n}{k_n h}$$
$$+ \frac{2A_n}{k_n{}^2 h^2} [1 - (1 + C k_n{}^2 h^2) \ell_n h P_n]\}, \tag{51}$$

and the first-order out-of-phase force coefficient C_{L1} may be obtained from (49) as

$$2a\omega h C_{L1} = [(1 - 2C + C^2 k_0{}^2 h^2) A_0 - 2B_0] \frac{P_0}{k_0 h}$$
$$+ \frac{2A_0}{k_0{}^2 h^2} [1 - (C k_0{}^2 h^2 - 1) \ell_0 h P_0]. \tag{52}$$

In obtaining (51) and (52), we have made use of relations (19) and (20). Both force coefficients C_{F1} and C_{L1} are also plotted in Figure 4 versus the wave-effect parameter C from $C = 0$ to $C = 0.5$. In the absence of wave effect, that is when $C = 0$, the out-of-phase force coefficient C_{L1} vanishes while the in-phase force coefficient C_{F1} has a negative value of -0.314. As C increases, C_{L1} increases monotonically from zero and C_{F1} also increases monotonically, however C_{F1} remains negative throughout the range of C from zero to 0.5. At $C = 0.5$, C_{L1} has a value of 0.225 and C_{F1} has a value of -0.044.

5. Conclusions

The effect of stratification on hydrodynamic pressures on a dam due to a horizontal, harmonic ground acceleration has been analyzed. It has been found that even for the constant-density solution the hydrodynamic pressure has an out-of-phase component as well as an in-phase component with respect to the given ground excitation. The out-of-phase component vanishes when the wave-effect parameter C, defined by $C = g/(\omega^2 h)$, vanishes. For any fixed height, the zeroth-order in-phase hydrodynamic pressure distribution on the dam decreases as C increases, while the magnitude of the zeroth-order out-of-phase pressure component increases with an increase of C. Thus the effect of surface waves is to reduce the in-phase pressure component and at the same time to increase the out-of-phase pressure component which is 90° lagging. For fixed values of C, the in-phase pressure coefficient increases with depth beneath the water surface and always attaines a maximum value at the bottom of the reservoir. On the other hand, the magnitude of the out-of-phase component increases towards the free surface with maximum values occurring on the free surface.

The first-order pressure distribution due to the stratification of the fluid in the reservoir also has two components, an in-phase component and an out-of-phase component. The out-of-phase component vanishes in the absence of surface waves. For any fixed height, both pressure components increase with an increase of C. For fixed values of C, the out-of-phase pressure increases monotonically towards the free surface, while the in-phase pressure decreases at first, then increases towards the free surface.

The total horizontal force on a dam due to harmonic ground excitations has also been presented. It is shown that both the zeroth-order solution and the first-order solution have in-phase and out-of-phase components, and the out-of-phase components become dominant when C is greater than 0.3.

Acknowledgement

The author is grateful to Mr. T. H. Huang for programming assistance. This work was sponsored by the National Science Foundation, under Grant PFR 77-16085A02.

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